

$^{10}\text{B}+\alpha$ states with chain-like structures in ^{14}N

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I investigate $^{10}\text{B}+\alpha$ cluster states of ^{14}N with a $^{10}\text{B}+\alpha$ cluster model. Near the α -decay threshold energy, I obtain $K^\pi = 3^+$ and $K^\pi = 1^+$ rotational bands having $^{10}\text{B}(3^+)+\alpha$ and $^{10}\text{B}(1^+)+\alpha$ components, respectively. I assign the band-head state of the $K^\pi = 3^+$ band to the experimental 3^+ at $E_x=13.19$ MeV of ^{14}N observed in α scattering reactions by ^{10}B and show that the calculated α -decay width is consistent with the experimental data. I discuss an α -cluster motion around the ^{10}B cluster and show that $^{10}\text{B}+\alpha$ cluster states contain significant components of a linear-chain 3α configuration, in which an α cluster is localized in the longitudinal direction around the deformed ^{10}B cluster.

I. INTRODUCTION

It is known that cluster structures appear in various nuclei including unstable nuclei (for instance, [1–5] and references therein.) For cluster states having an α cluster around a core nucleus, well-known examples are $^{16}\text{O}+\alpha$ states in ^{20}Ne and $^{12}\text{C}+\alpha$ states in ^{16}O [6]. Recent experimental and theoretical studies have revealed many cluster resonances in highly excited states near the α -decay threshold also in unstable nuclei, for instance, $^A\text{He}+\alpha$ states in Be isotopes [1, 4, 7–26], $^{10}\text{Be}+\alpha$ states in ^{14}C [27–30, 54], $^{14}\text{C}+\alpha$ states in ^{18}O and their mirror states [31–40], $^{18}\text{O}+\alpha$ states in ^{22}Ne [38–45].

Multi- α cluster states such as cluster gas and linear-chain states of $n\alpha$ systems are also interesting topics. The α -cluster gas was proposed by Tohsaki *et al.* to describe 3α cluster structure of $^{12}\text{C}(0_2^+)$ [46] and extended to excited states of ^{12}C and other nuclei [47–49]. The linear-chain $n\alpha$ state was originally proposed for $^{12}\text{C}(0_2^+)$ by Morinaga in the 1950-60s [50, 51]. However, in the 1970s, this picture was excluded at least for $^{12}\text{C}(0_2^+)$ having a larger α -decay width than the one expected from the linear-chain structure [52]. In spite of many discussions for several decades, existence of linear-chain $n\alpha$ states have yet been confirmed and it is still an open problem to be solved. It is naively expected that the linear-chain configuration is not favored in an $n\alpha$ system because it costs much kinetic energy to keep α clusters in a row. It means that some mechanism is necessary to form the linear-chain structure. In progress of physics of unstable nuclei since the 1990s, it was proposed for neutron-rich C isotopes that excess neutrons may stabilize the linear-chain structure [1, 8]. Itagaki *et al.* analyzed stability of a 3α -chain configuration surrounded by excess neutrons in molecular orbitals against the bending motion and suggested that the linear-chain structure can be stable in ^{16}C but unstable in ^{12}C and ^{14}C [53]. More recently, Suhara and the author predicted a rotational band with a linear 3α chain configuration in excited states of ^{14}C near the α -decay threshold [54]. They pointed out that the orthogonal condition to lower states is important for the stability of the linear-chain structure. The linear-chain structure is expected to be more favored in high spin states because of stretching effect in rotating systems as suggested in ^{15}C [1] and ^{16}O [55].

According to analysis in Refs. [54, 56], linear-chain states of ^{14}C are found to have a $2\alpha + 2n$ correlation and are interpreted as $^{10}\text{Be}+\alpha$ structures, where the ^{10}Be cluster is a prolately deformed state containing a 2α core and an additional α cluster is located in the longitudinal direction of the ^{10}B cluster. Similarly, the linear-chain state of ^{15}C suggested in Ref. [1] also shows a $^{11}\text{Be}+\alpha$ cluster structure with a prolately deformed ^{11}Be cluster and an α cluster in the longitudinal direction. It means that, the linear-chain states in these neutron-rich C tend to have the 2α correlation, and therefore 3α linear-chain structures are expected to be found in $\text{Be}+\alpha$ cluster states.

In this paper, I focus on $^{10}\text{B}+\alpha$ cluster states in excited states of ^{14}N . In experimental energy levels of ^{14}N near the α -decay threshold, $J^\pi = 3^+$ and 1^+ resonances were observed by α elastic scattering by ^{10}B [57]. These resonances are expected to be $^{10}\text{B}+\alpha$ cluster states because of significant α -decay widths. In analogy to $^{10}\text{Be}+\alpha$ cluster states, it is interesting to investigate whether $^{10}\text{B}+\alpha$ cluster states with the dominant linear-chain structure exist. The ground state (3^+) and the first excited state (1^+) of ^{10}B can be described by the deformed state with a 2α core surrounded by pn as discussed in Refs. [7, 58]. If a $^{10}\text{B}+\alpha$ cluster state has an α cluster in the longitudinal direction of the deformed ^{10}B cluster, the $^{10}\text{B}+\alpha$ cluster state can be interpreted as a kind of linear-chain state that contains dominantly 3α clusters arranged in a row.

My aim is to study $^{10}\text{B}+\alpha$ cluster states of ^{14}N and discuss 3α configurations, in particular, the linear-chain component in $^{10}\text{B}+\alpha$ cluster states. I calculate $^{10}\text{B}(3^+) \otimes L_\alpha$ and $^{10}\text{B}(1^+) \otimes L_\alpha$ components and evaluate partial α -decay widths of $^{10}\text{B}+\alpha$ cluster states. To discuss stability of the linear-chain $^{10}\text{B}+\alpha$ structure, I analyze angular motion of an α cluster around the deformed ^{10}B cluster, i.e., rotation of the ^{10}B cluster.

This paper is organized as follows. In Sec. II, I explain the formulation of the present $^{10}\text{B}+\alpha$ cluster model. In Sec. III, calculated positive-parity states and $E2$ transition strengths of ^{14}N are shown. I discuss α cluster motion

around $^{10}\text{B}(3^+)$ and $^{10}\text{B}(1^+)$ in Sec. IV. Finally, a summary is given in Sec. V.

II. FORMULATION OF THE $^{10}\text{B}+\alpha$ CLUSTER MODEL

A. Description of the ^{10}B cluster

For the ^{10}B cluster in the present $^{10}\text{B}+\alpha$ cluster model, I adopt a $2\alpha + (pn)$ wave function which can reasonably describe features of the ground ($J^\pi = 3^+$) and first excited (1^+) states of ^{10}B as discussed in Ref. [58]. The $2\alpha + (pn)$ wave function is given by a three-body cluster wave function, where α clusters and a dinucleon (pn) cluster are written by $(0s)^4$ and $(0s)^2$ harmonic oscillator configurations, respectively, as

$$\Phi_{2\alpha+pn}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3) = \mathcal{A}\{\Phi_\alpha(\mathbf{R}_1)\Phi_\alpha(\mathbf{R}_2)\Phi_{pn}(\mathbf{R}_3)\}, \quad (1)$$

$$\Phi_\alpha(\mathbf{R}) = \psi_{p\uparrow}(\mathbf{R})\psi_{p\downarrow}(\mathbf{R})\psi_{n\uparrow}(\mathbf{R})\psi_{n\downarrow}(\mathbf{R}), \quad (2)$$

$$\Phi_{pn}(\mathbf{R}) = \psi_{p\uparrow}(\mathbf{R})\psi_{n\uparrow}(\mathbf{R}), \quad (3)$$

$$\psi_\sigma(\mathbf{R}) = \varphi_{0s}(\mathbf{R})\chi_\sigma, \quad (4)$$

where \mathcal{A} is the antisymmetrizer for all nucleons and $\varphi_{0s}(\mathbf{R})$ is the spatial part of the single-particle wave function of the $0s$ orbit around \mathbf{R} ;

$$\varphi_{0s}(\mathbf{R}) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\{-\nu(\mathbf{r} - \mathbf{R})^2\}, \quad (5)$$

and χ_σ is the spin-isospin wave function for $\sigma = p\uparrow, p\downarrow, n\uparrow$, and $n\downarrow$. For the ^{10}B cluster, I set 2 α clusters in the z direction as $\mathbf{R}_1 - \mathbf{R}_2 = (0, 0, d_{2\alpha})$ with $d_{2\alpha} = 3$ fm and a spin-aligned pn cluster on the x - y plane at the distance d from the 2α center as $\mathbf{R}_3 - (\mathbf{R}_1 + \mathbf{R}_2)/2 = (d\cos\phi, d\sin\phi, 0)$. I write the ^{10}B wave function localized around $\mathbf{X}_B \equiv (4\mathbf{R}_1 + 4\mathbf{R}_2 + 2\mathbf{R}_3)/10$ as $\Phi_{^{10}\text{B}}(\mathbf{X}_B; d, \phi)$ with the center position \mathbf{X}_B and the distance and angle parameters, d and ϕ , for the pn cluster position. In the $^{10}\text{B}+\alpha$ cluster model, I superpose the ^{10}B wave functions with $d = 1, 2$ (fm) and $\phi_j = \frac{\pi}{4}(j - 0.5)$ ($j = 1, \dots, 8$). Parity (π) and K (I_z) projections of the subsystem ^{10}B can be approximately done by the ϕ_j summation;

$$\Phi_{^{10}\text{B}(I_z^\pi)}(\mathbf{X}_B; d) = \sum_j c_j \Phi_{^{10}\text{B}}(\mathbf{X}_B; d, \phi_j) \quad (6)$$

with $c_j = \exp(i(I_z - 1)\phi_j)$ and $\pi = (-1)^{I_z-1}$. Here, I_z is the z component of the total angular momentum \mathbf{I} of ^{10}B and is given by a sum of the aligned intrinsic spin $S_z = 1$ and the orbital ϕ rotation of the pn cluster. It is clear that ϕ_j superposition with given coefficients c_j is equivalent to I_z mixing.

B. ^{14}N wave function in the $^{10}\text{B} + \alpha$ model

A $^{10}\text{B} + \alpha$ wave function is written using the ^{10}B wave function $\Phi_{^{10}\text{B}}(\mathbf{X}_B; d, \phi)$ and the α -cluster wave function $\Phi_\alpha(\mathbf{X}_\alpha)$ as

$$\Phi_{^{10}\text{B}+\alpha}(D_\alpha, \theta_\alpha; d, \phi) = \mathcal{A}\{\Phi_{^{10}\text{B}}(\mathbf{X}_B; d, \phi)\Phi_\alpha(\mathbf{X}_\alpha)\}, \quad (7)$$

where $\mathbf{X}_\alpha - \mathbf{X}_B = (D_\alpha \sin\theta_\alpha, 0, D_\alpha \cos\theta_\alpha)$. Here the distance D_α and the angle θ_α indicate the α -cluster position relative to the deformed ^{10}B cluster (see Fig. 1). The center of mass position is taken to be $4\mathbf{X}_\alpha + 10\mathbf{X}_B = 0$ so as to decouple the center of mass motion and the intrinsic wave function. Wave functions for the J_n^π states of ^{14}N are expressed by superposition of the J^π -projected wave functions as

$$\Psi_{^{14}\text{N}(J_n^\pi)} = \sum_K \sum_{D_\alpha, \theta_\alpha} \sum_{d, \phi} C(K, D_\alpha, \theta_\alpha, d, \phi) \hat{P}_{MK}^{J_n^\pi} \Phi_{^{10}\text{B}+\alpha}(D_\alpha, \theta_\alpha; d, \phi), \quad (8)$$

where $\hat{P}_{MK}^{J_n^\pi}$ is the parity and total angular momentum projection operator. Coefficients $C(K, D_\alpha, \theta_\alpha, d, \phi)$ are determined by diagonalizing Hamiltonian and norm matrices. I take $D_\alpha = \{2, \dots, 6\}$ (fm), $\theta_\alpha = \{0, \pi/4, \pi/2\}$, $d = \{1, 2\}$ (fm), and $\phi = \frac{\pi}{4}(j - 0.5)$ ($j = 1, \dots, 8$). In the present paper, I calculate positive-parity ($\pi = +$) states of ^{14}N .

In Eq. (8), coupling of \mathbf{I} (the spin of the ^{10}B cluster) and \mathbf{L}_α (the orbital angular momentum of the α cluster relative to the ^{10}B cluster) is implicitly described by the J^π projection, K mixing, and θ_α , ϕ summations. As shown in Fig. 1, \mathbf{L}_α couples with \mathbf{I} to the total angular momentum $\mathbf{J} = \mathbf{L}_\alpha + \mathbf{I}$. The z component, $J_z = I_z + L_{\alpha z}$, is the so-called K quantum. Strictly speaking, $L_\alpha = 0, 2$ (S, D -wave) mixing is approximately taken into account by the summation of $\theta_\alpha = \{0, \pi/4, \pi/2\}$ but higher $L_\alpha (\geq 4)$ mixing can not be controlled in the present calculation because of the finite number of mesh points for θ_α .

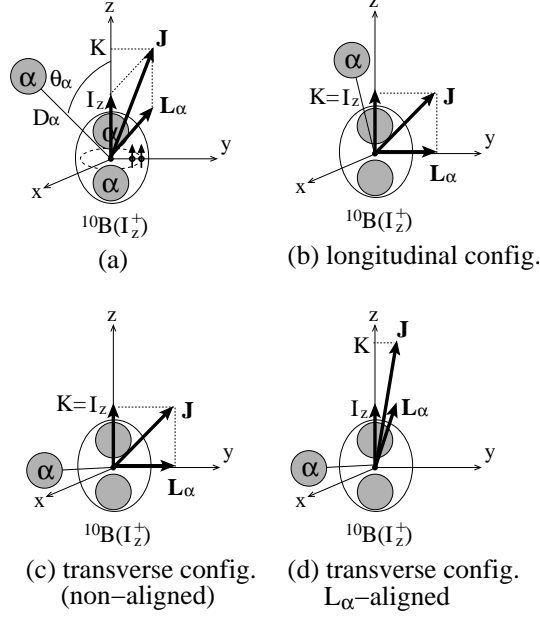


FIG. 1: Schematic figures for $^{10}\text{B} + \alpha$ configurations. (a) Parameters of the $^{10}\text{B} + \alpha$ cluster model. (b) Longitudinal configuration for $\theta_\alpha \sim 0$, (c) transverse configuration for $\theta_\alpha \sim \pi/2$ without L_α alignment, and (d) L_α -aligned transverse configuration.

C. Overlap function and α -cluster probability

In order to evaluate $^{10}\text{B}(3^+) \otimes L_\alpha$ and $^{10}\text{B}(1^+) \otimes L_\alpha$ components at a channel radius (D_α), I consider the $L_\alpha L_{\alpha z}$ projected $^{10}\text{B}(I_z^\pi) + \alpha$ wave function,

$$|J^\pi K; ^{10}\text{B}(I_z^\pi); D_\alpha, L_\alpha L_{\alpha z}\rangle = n_0 \sum_{\theta_\alpha} \omega(\theta_\alpha) Y_{L_{\alpha z}}^{L_\alpha}(\theta_\alpha) \hat{P}_{MK}^{J^\pi} \Phi_{^{10}\text{B}(I_z^\pi) + \alpha}(D_\alpha, \theta_\alpha), \quad (9)$$

$$\Phi_{^{10}\text{B}(I_z^\pi) + \alpha}(D_\alpha, \theta_\alpha) = \mathcal{A} \{ \Phi_{^{10}\text{B}(I_z^\pi)}(\mathbf{X}_B; d) \Phi_\alpha(\mathbf{X}_\alpha) \}, \quad (10)$$

with $I_z = \{1, 3\}$, $\pi = +$, $K = I_z + L_{\alpha z}$, $\mathbf{X}_\alpha - \mathbf{X}_B = (D_\alpha \sin \theta_\alpha, 0, D_\alpha \cos \theta_\alpha)$, and $4\mathbf{X}_\alpha + 10\mathbf{X}_B = 0$. Y_μ^λ is the spherical harmonics. $\Phi_{^{10}\text{B}(I_z^\pi) + \alpha}(D_\alpha, \theta_\alpha)$ is the wave function for the α cluster at $(D_\alpha, \theta_\alpha)$ around the I_z projected ^{10}B cluster, for which I fix $d = 2$ fm in the present analysis. n_0 is determined from the normalization condition $\langle JK; ^{10}\text{B}(I_z^\pi); D_\alpha, L_\alpha L_{\alpha z} | JK; ^{10}\text{B}(I_z^\pi); D_\alpha, L_\alpha L_{\alpha z} \rangle = 1$. In Eq. (9), the $L_\alpha L_{\alpha z}$ projection is approximately performed by the summation $\theta_\alpha = \frac{\pi}{N_\theta} i$ ($i = 0, \dots, N_\theta$) with the weight function $\omega(\theta_\alpha) = \int_{\min[\theta_\alpha - \pi/2N_\theta, 0]}^{\max[\theta_\alpha + \pi/2N_\theta, \pi]} \sin \theta d\theta$. I perform only $L_\alpha = 0, 2$ projections because $L_\alpha \geq 4$ projections are not possible for the present $N_\theta = 4$ case. I calculate the squared overlap of the ^{14}N wave function with the above wave function, $|\langle JK; ^{10}\text{B}(I_z^\pi); D_\alpha, L_\alpha L_{\alpha z} | \Psi_{^{14}\text{N}(J_n^\pi)} \rangle|^2$. Assuming that the 3_1^+ and 1_1^+ states of the ^{10}B cluster are approximately described by the I_z projected ^{10}B wave functions, $^{10}\text{B}(I_z^\pi = 3^+)$ and $^{10}\text{B}(I_z^\pi = 1^+)$, respectively, I approximately estimate the $^{10}\text{B}(I^\pi) \otimes (L_\alpha = 0, 2)$ components as

$$P_{^{10}\text{B}(I^\pi) \otimes L_\alpha}(D_\alpha) \approx \sum_{L_{\alpha z}} |\langle JK | I I_z L_\alpha L_{\alpha z} \rangle \langle JK; ^{10}\text{B}(I_z); D_\alpha, L_\alpha L_{\alpha z} | \Psi_{^{14}\text{N}(J_n^\pi)} \rangle|^2 \quad (11)$$

with $I_z = I$, where $\langle JK | I I_z L_\alpha L_{\alpha z} \rangle$ is the Clebsch-Gordan coefficient.

I also calculate α -cluster probability at $(D_\alpha, \theta_\alpha)$ around the I_z projected ^{10}B cluster as

$$P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha) = |\langle JK; D_\alpha, \theta_\alpha; ^{10}\text{B}(I_z^\pi) | \Psi_{^{14}\text{N}(J_n^\pi)} \rangle|^2, \quad (12)$$

$$|JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha\rangle = n_0 \hat{P}_{MK}^{J+} \Phi_{^{10}\text{B}(I_z^\pi)+\alpha}(D_\alpha, \theta_\alpha). \quad (13)$$

The probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ is useful to discuss geometric configurations of 3α clusters in $^{10}\text{B}+\alpha$ cluster states in the strong coupling picture. For instance, $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ for $\theta_\alpha \sim 0$ means the component of the “longitudinal” configuration, where the α cluster is localized in the longitudinal direction of the deformed $^{10}\text{B}(I_z^\pi)$ cluster. This configuration corresponds to the linear-chain structure as 3 α clusters are arranged in a row as shown in Fig. 1(b). Because of the axial symmetry, the longitudinal configuration contains only $K = I_z$ ($L_{\alpha z} = 0$) component. For $\theta_\alpha \sim \pi/2$, $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ indicates the component of the “transverse configuration” for the α cluster in the transverse direction of the deformed $^{10}\text{B}(I_z^\pi)$ cluster. The transverse configuration contains $K \neq I_z$ components corresponding to the alignment of \mathbf{L}_α to the spin of the pn -cluster (I_z) in the ^{10}B cluster as well as the $K = I_z$ component (see Fig. 1(b) and (c)).

III. RESULTS

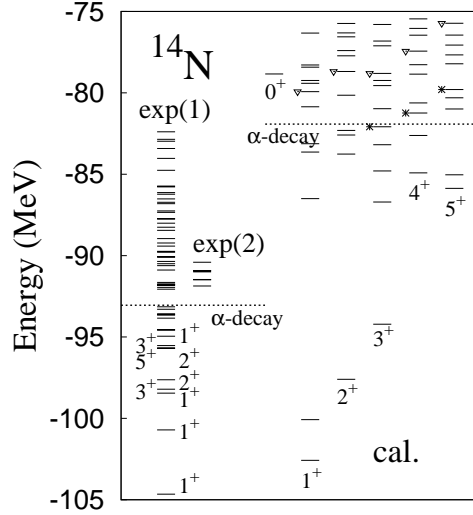


FIG. 2: Positive-parity energy levels of ^{14}N obtained by the $^{10}\text{B}+\alpha$ cluster model compared with experimental levels taken from [61]. $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ band and those in the $K^\pi = 1^+$ band are labeled by asterisk and down-triangle symbols, respectively.

I adopt the two-body effective nuclear interactions used in Ref. [58] which are adjusted to describe low-lying energy levels of ^{10}B . Namely, I use the Volkov central force [59] with the Bartlett, Heisenberg, and Majorana parameters $b = h = 0.06$ and $m = 0.60$ and the G3RS spin-orbit force [60] with the strength $u_I = -u_{II} = 1300$ MeV, and the Coulomb force approximated by seven Gaussians. Using these interactions, Energies of ^{10}B are obtained to be -54.3 MeV for the ground state (3^+) and -53.4 MeV for the first excite state (1^+) with the $2\alpha + pn$ cluster model by superposing $\sum_{I_z, d} \hat{P}_{MI_z}^{I\pi} \Phi_{^{10}\text{B}}(\mathbf{X}_B = 0; d, \phi = 0)$ with $d = 1, 2$ (fm). Though the calculation underestimates the experimental binding energy (64.75 MeV), it reproduces the spin parity of the ground state ($^{10}\text{B}(3_{\text{g.s.}}^+)$), and also the calculated excitation energy $E_x = 0.9$ MeV of the 1^+ state reasonably agrees to the experimental value $E_x = 0.72$ MeV for $^{10}\text{B}(1_1^+)$.

Using the $^{10}\text{B}+\alpha$ cluster wave function in Eq. (8), I calculate positive-parity states of ^{14}N . Properties of the ground state $^{14}\text{N}(1_{\text{g.s.}}^+)$ are reasonably reproduced by the present calculation. Namely, the binding energy B.E.=102.6 MeV, the magnetic moment $\mu = 0.36$ (μ_N), and the electric quadrupole moment $Q = 2.4$ (efm^2) of $^{14}\text{N}(1_{\text{g.s.}}^+)$, obtained by the present calculation reasonably agree to the experimental data (B.E.=104.66 MeV, $\mu = 0.4038$ (μ_N), and $Q = 1.93(8)$ (efm^2)). The calculated energy spectra are shown in Fig. 2. The α -decay threshold is much higher

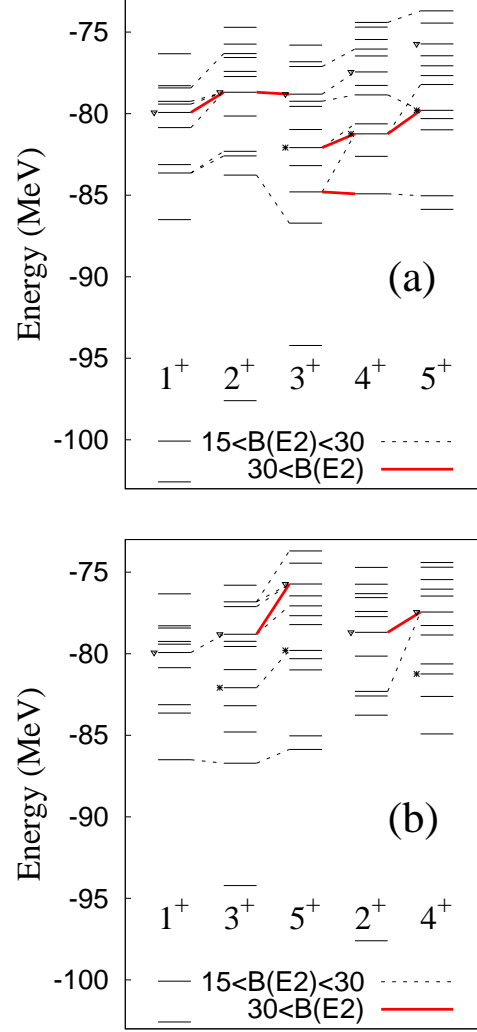


FIG. 3: (Color online) $E2$ transition strengths calculated by the $^{10}\text{B} + \alpha$ cluster model for (a) $J^+ \rightarrow J^+ - 1$ and (b) $J^+ \rightarrow J^+ - 2$ transitions with $B(E2) \geq 15 \text{ e}^2 \text{ fm}^4$. Asterisk and down-triangle symbols show $^{10}\text{B} + \alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, respectively.

in the present calculation than the experimental threshold. In other words, the ground and some low-lying states of ^{14}N show too deep binding from the α -decay threshold compared with the experimental data. The significant overestimation of the α -decay threshold is a general problem in microscopic calculations with density-independent two-body effective interactions as found for ^{14}C and O isotopes [6, 32, 54]. One of the origins of this problem is a difficulty in reproducing systematics of binding energies in a wide mass-number region with such effective interactions. In this paper, I mainly investigate $^{10}\text{B} + \alpha$ cluster states near the α -decay threshold and discuss their features. In the calculated energy levels near the threshold, I obtain several excited states having significant component of a spatially developed α cluster around the ^{10}B cluster. From remarkable $E2$ transitions, I assign the $^{10}\text{B} + \alpha$ cluster states to a $K^\pi = 3^+$ band of $J^\pi = 3^+, 4^+, \text{ and } 5^+$ states, and a $K^\pi = 1^+$ band of $J^\pi = 1^+, 2^+, 3^+, 4^+, \text{ and } 5^+$ states. The former and the latter bands are shown by asterisk and down-triangle symbols in Fig. 2. The $K^\pi = 3^+$ band has the significant $^{10}\text{B}(3^+) + \alpha$ component, whereas the $K^\pi = 1^+$ band contains the $^{10}\text{B}(1^+) + \alpha$ component. More details of structure of these states are discussed in the next section.

Figure 3 shows $E2$ transitions with $B(E2) \geq 15 \text{ e}^2 \text{ fm}^4$ for $J \rightarrow J - 1$ and $J \rightarrow J - 2$ transitions. In-band transitions for the $K^\pi = 3^+$ and $\pi = 1^+$ $^{10}\text{B} + \alpha$ bands are rather strong because of the developed cluster structures, though $E2$ strengths are somewhat fragmented into neighboring states.

IV. DISCUSSION

$^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands have maximum amplitudes of α -cluster probability around $D_\alpha = 5$ fm. In this section, I focus on angular motion of the α cluster at $D_\alpha = 5$ fm. I first investigate the angular momentum coupling of the α -cluster (L_α) and the ^{10}B cluster (I) in a weak coupling picture and estimate α -decay widths. Then, I discuss geometric configurations of $^{10}\text{B}+\alpha$ cluster states in the strong coupling picture by analyzing θ_α -dependence of the α -cluster probability around the deformed ^{10}B cluster.

A. D_α -fixed calculation

In the present calculation, radial motion of the α cluster is described by superposing $^{10}\text{B}+\alpha$ wave functions for $D_\alpha = 2, \dots, 6$ fm. Instead of the full model space in Eq. (8) including $D_\alpha = 2, \dots, 6$ fm wave functions, I also perform a similar calculation using the D_α -fixed model space

$$\Psi_{^{14}\text{N}(J_n^\pi)}^{D_\alpha=5} = \sum_K \sum_{\theta_\alpha} \sum_{d,\phi} C(K, \theta_\alpha, d, \phi) \hat{P}_{MK}^{J_\pi} \Phi_{^{10}\text{B}+\alpha}(D_\alpha, \theta_\alpha; d, \phi), \quad (14)$$

where I fix $D_\alpha = 5$ fm and take $\theta_\alpha = \{0, \pi/8, \pi/4, 3\pi/8, \pi/2\}$, $d = \{1, 2\}$ (fm), and $\phi = \frac{\pi}{4}(j - 0.5)$ ($j = 1, \dots, 8$). In the $D_\alpha = 5$ fm fixed calculation, I find the states near the threshold energy corresponding to $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, but do not obtain lower states below the threshold because of the truncation of the model space. Energy levels of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained with the full and D_α -fixed calculations are shown in Fig. 4. The calculated energies are measured from the α -decay threshold. The experimental levels observed by α elastic scattering by ^{10}B are also shown in the figure. The level structures of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands are essentially consistent between the full and D_α -fixed calculations, though about 2 MeV global shift is found for the $K^\pi = 3^+$ band between two calculations.

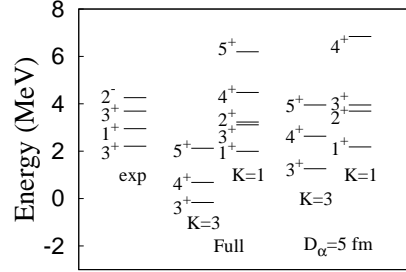


FIG. 4: Energies of $^{10}\text{B}+\alpha$ cluster states obtained by the full and D_α -fixed calculations and those observed by the experiment of $^{10}\text{B}(\alpha, \alpha)^{10}\text{B}$ reactions [57]. Energies are measured from the α -decay threshold.

B. α -cluster probability and α -decay widths

In Table I, I show L_α components ($P_{^{10}\text{B}(I^\pi) \otimes L_\alpha}$ in Eq. (11)) at $D_\alpha = 5$ fm coupled with $^{10}\text{B}(3^+)$ and $^{10}\text{B}(1^+)$ in $^{10}\text{B}+\alpha$ cluster states obtained by the full and D_α -fixed calculations. In the result of the D_α -fixed calculation, $K^\pi = 3^+$ band states are dominated by the $^{10}\text{B}(3^+) \otimes L_\alpha$ component, whereas $K^\pi = 1^+$ band states contain dominantly the $^{10}\text{B}(1^+) \otimes L_\alpha$ component. In the result of the full calculation, the $K^\pi = 3^+$ and $K^\pi = 1^+$ band states still contain significant $^{10}\text{B}(3^+) \otimes L_\alpha$ and $^{10}\text{B}(1^+) \otimes L_\alpha$ components, respectively, except for the $1^+(K^\pi = 1^+)$ state, though the absolute amplitude of the dominant component decreases because of radial motion and state mixing. The $1^+(K^\pi = 1^+)$ state obtained by the full calculation shows a feature quite different from that obtained by the D_α -fixed calculation. In the D_α -fixed calculation, the $1^+(K^\pi = 1^+)$ state is approximately described by the pure $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ state, where the orbital angular momentum (L_α) of the α cluster weakly couples to the spin (I) of the ^{10}B cluster. However, in the full calculation, the $1^+(K^\pi = 1^+)$ state does not show the weak coupling feature but has $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$, $^{10}\text{B}(1^+) \otimes (L_\alpha = 2)$, and $^{10}\text{B}(3^+) \otimes (L_\alpha = 2)$ components with the same order showing a strong coupling feature.

Figure 5 shows L_α components ($P_{10B(I\pi)\otimes L_\alpha}$) at $D_\alpha = 5$ fm of J^π states in the ^{14}N spectra obtained by the full calculation. The $^{10}\text{B}(3^+)+(L_\alpha = 0)$ and $^{10}\text{B}(3^+)+(L_\alpha = 2)$ components concentrate at the $3^+(K^\pi = 3^+)$ and $4^+(K^\pi = 3^+)$ states, respectively, though the components are fragmented into other states. The $5^+(K^\pi = 3^+)$ state shows rather strong state mixing. The $^{10}\text{B}(1^+)+(L_\alpha = 2)$ component concentrates at the $2^+(K^\pi = 1^+)$ and $3^+(K^\pi = 1^+)$ states, whereas, the $^{10}\text{B}(1^+)+(L_\alpha = 0)$ component feeds lower 1^+ states of ^{14}N .

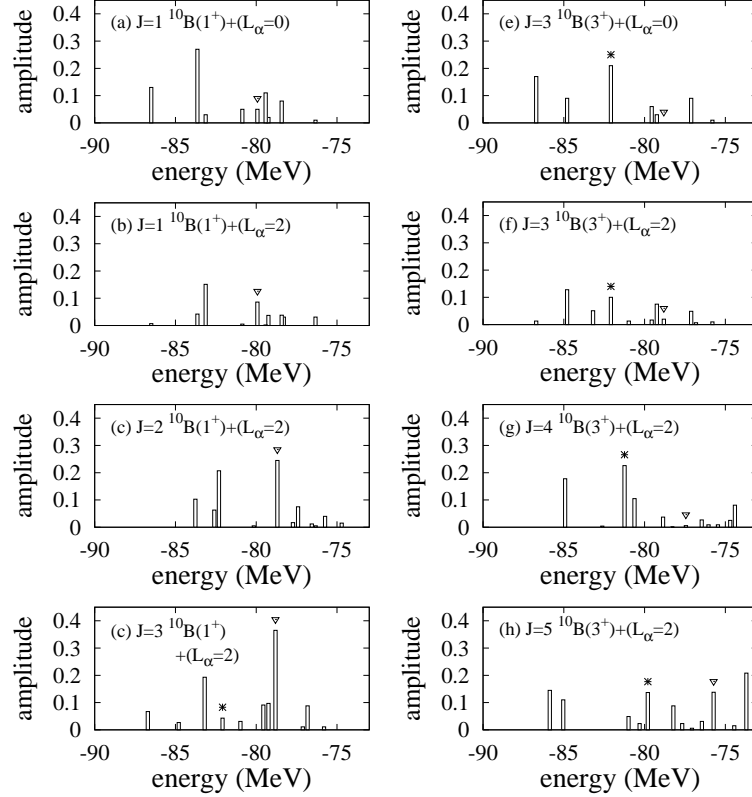


FIG. 5: $^{10}\text{B}(I^\pi) \otimes (L_\alpha = 0, 2)$ components, $P_{10B(I\pi)\otimes L_\alpha}(D_\alpha = 5\text{fm})$, in positive-parity states of ^{14}N obtained by the $^{10}\text{B}+\alpha$ -cluster model. Asterisk and down-triangle symbols show $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, respectively.

In the experiment of $^{10}\text{B}(\alpha, \alpha)^{10}\text{B}$ reactions [57], the 3^+ state at $E_r = 1.58$ MeV ($E_x = 13.19$ MeV) with the width $\Gamma = 0.065$ MeV is strongly populated. In the analysis of Ref. [57], this state is described well by the dominant (almost 100%) S -wave α -decay indicating the significant $^{10}\text{B}(3^+) \otimes (L_\alpha = 0)$ component of the 3^+ state. The 1^+ state at $E_r = 2.11$ MeV ($E_x = 13.72$ MeV) is weakly populated in $^{10}\text{B}(\alpha, \alpha)^{10}\text{B}$ reactions, whereas its α -decay into the first excited state of $^{10}\text{B}(1^+)$ was observed in $^{10}\text{B}(\alpha, \alpha')^{10}\text{B}$ reactions [62]. These experiments suggest that the 1^+ state would contain $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ and $^{10}\text{B}(3^+) \otimes (L_\alpha = 2)$ components.

From the experimental α -decay properties, I tentatively assign the theoretical $3^+(K^\pi = 3^+)$ and $1^+(K^\pi = 1^+)$ states having $^{10}\text{B}+\alpha$ cluster structures to the experimental 3^+ ($E_r^{\text{exp}} = 1.58$ MeV) and 1^+ ($E_r^{\text{exp}} = 2.11$ MeV) states, though the band-head energies $E_r(3^+; K^\pi = 3^+) = -0.2$ MeV and $E_r(1^+; K^\pi = 1^+) = 2.0$ MeV obtained by the full calculation do not necessarily agree to the experimental energies (see Fig. 4). I estimate partial α -decay widths for $B(I^\pi) \otimes L_\alpha$ channels from $P_{10B(I\pi)\otimes L_\alpha}(D_\alpha = a)$ (a is the channel radius) as follows. Using the approximate evaluation of the reduced width amplitude proposed in Ref. [63], the reduced width $\gamma_\alpha^2(a)$ is calculated as

$$\gamma_\alpha^2(a) = \frac{\hbar^2}{2\mu a} \left(\frac{\nu}{2\pi} \frac{A_1 A_2}{A_1 + A_2} \right)^{1/2} P_{10B(I\pi)\otimes L_\alpha}(D_\alpha = a), \quad (15)$$

and the partial α -decay width $\Gamma_{10B(I\pi)+\alpha}$ for $L_\alpha = l$ is calculated as

$$\Gamma_{10B(I\pi)+\alpha} = 2P_l(a)\gamma_\alpha^2(a), \quad (16)$$

$$P_l(a) = \frac{ka}{F_l^2(ka) + G_l^2(ka)}, \quad (17)$$

where $k = \sqrt{2\mu E}/\hbar$, and F_l and G_l are the regular and irregular Coulomb functions, respectively. Here I use the momentum k of the energy $E = E_r^{(\text{adjust})}$ which is phenomenologically adjusted to the experimental energy position because it is difficult to quantitatively predict the energy position in the present calculation. Namely, I adjust the band-head energies of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands to the experimental energy positions $E_r^{\text{exp}}(3^+) = 1.58$ MeV and $E_r^{\text{exp}}(1^+) = 2.11$ MeV, by a constant shift for each band as

$$E_r^{(\text{adjust})}(J^+; K^\pi = 3^+) = E_r(J^+; K^\pi = 3^+) - E_r(3^+; K^\pi = 3^+) + E_r^{\text{exp}}(3^+), \quad (18)$$

$$E_r^{(\text{adjust})}(J^+; K^\pi = 1^+) = E_r(J^+; K^\pi = 1^+) - E_r(1^+; K^\pi = 1^+) + E_r^{\text{exp}}(1^+). \quad (19)$$

Calculated partial α -decay widths are shown in Table I. I calculate widths for $L_\alpha = 0$ and $L_\alpha = 2$ channels. α -decay widths obtained by the full calculation are several times smaller than those obtained by the D_α -fixed calculation because of the suppression of the α -cluster probability as shown previously. As a result, the α -decay width of the $3^+(K^\pi = 3^+)$ state reduces to be $\Gamma_\alpha = 0.05$ MeV with the dominant $^{10}\text{B}(3^+) \otimes (L_\alpha = 0)$ decay, which is quantitatively consistent with the experimental observation ($\Gamma_\alpha \sim \Gamma = 0.065(10)$ MeV) [57]. For the $1^+(K^\pi = 1^+)$ state, I obtain a small α -decay width $\Gamma_\alpha = 0.01$ MeV with the dominant $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ decay. This result seems consistent with the weak population in the α elastic scattering [57] and the fact that the 1^+ state was observed in $^{10}\text{B}(\alpha, \alpha'\gamma)^{10}\text{B}$ reaction [62]. However, experimental information of partial α -decay widths is not enough to confirm the present assignment of the $1^+(K^\pi = 1^+)$ state. The calculated α -decay width is much smaller than the experimental total width, $\Gamma = 0.16(2)$ MeV, of the 1^+ state at 2.11 MeV. I should comment that, because the $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ component is fragmented into neighboring states as shown in Fig. 5, an effectively large width could be observed for the $1^+(K^\pi = 1^+)$ state.

TABLE I: $^{10}\text{B}(I^\pi) \otimes (L_\alpha = 0, 2)$ components, $P_{^{10}\text{B}(I^\pi) \otimes L_\alpha}(D_\alpha = 5 \text{ fm})$, of $^{10}\text{B} + \alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained by the full and D_α -fixed calculations.

J^π	$P_{^{10}\text{B}(3^+) \otimes L_\alpha}$		$P_{^{10}\text{B}(1^+) \otimes L_\alpha}$	
	$L_\alpha = 0$	$L_\alpha = 2$	$L_\alpha = 0$	$L_\alpha = 2$
full cal.				
$3^+(K^\pi = 3^+)$	0.21	0.10		0.04
$4^+(K^\pi = 3^+)$		0.23		
$5^+(K^\pi = 3^+)$		0.14		
$1^+(K^\pi = 1^+)$		0.03	0.05	0.09
$2^+(K^\pi = 1^+)$		0.02		0.25
$3^+(K^\pi = 1^+)$	0.00	0.02		0.37
$4^+(K^\pi = 1^+)$		0.01		
$5^+(K^\pi = 1^+)$		0.14		
D_α -fixed cal.				
$3^+(K^\pi = 3^+)$	0.57	0.25		0.01
$4^+(K^\pi = 3^+)$		0.73		
$5^+(K^\pi = 3^+)$		0.75		
$1^+(K^\pi = 1^+)$		0.02	0.89	0.05
$2^+(K^\pi = 1^+)$		0.01		0.78
$3^+(K^\pi = 1^+)$	0.10	0.13		0.74
$4^+(K^\pi = 1^+)$		0.00		

C. Angular motion of the α cluster around the deformed ^{10}B cluster

I here discuss angular motion of the α -cluster around the deformed ^{10}B cluster by analyzing θ_α dependence of α -cluster probabilities. Discussions in this section are based on the strong coupling picture, which is somehow different from the previous discussion based on the L_α decomposition in the weak coupling picture. I show energies of $\Phi_{^{10}\text{B}(I_z^\pi) + \alpha}(D_\alpha, \theta_\alpha)$, in which the α cluster is localized at $(D_\alpha, \theta_\alpha)$ around the I_z projected ^{10}B cluster. In Fig. 5, intrinsic energies before parity and angular-momentum projections of $\Phi_{^{10}\text{B}(I_z^\pi) + \alpha}(D_\alpha, \theta_\alpha)$ for $I_z^\pi = 3^+$ and 1^+ are

TABLE II: Partial α -decay widths of $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained by the full and D_α -fixed calculations. Energies of the band-head states of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands are adjusted to the experimental resonance energies of the 3^+ state at 1.58 MeV and the 1^+ state at 2.11 MeV. The sum ($\Gamma_{^{10}\text{B}+\alpha}(L_\alpha \leq 2)$) of partial widths of decay channels $^{10}\text{B}(3^+) \otimes (L_\alpha \leq 2)$ and $^{10}\text{B}(1^+) \otimes (L_\alpha \leq 2)$ is also shown. The unit is MeV.

J^π	$E_r^{(\text{adjust})}$	$\Gamma_{^{10}\text{B}(3^+)+\alpha}$		$\Gamma_{^{10}\text{B}(1^+)+\alpha}$		$\Gamma_{^{10}\text{B}(3^+)+\alpha}(L_\alpha \leq 2)$
		$L_\alpha = 0$	$L_\alpha = 2$	$L_\alpha = 0$	$L_\alpha = 2$	
full cal.						
$3^+(K^\pi = 3^+)$	1.58	0.04	0.00		0.00	0.05
$4^+(K^\pi = 3^+)$	2.43		0.06			0.06
$5^+(K^\pi = 3^+)$	3.87		0.16			0.16
$1^+(K^\pi = 1^+)$	2.11		0.00	0.01	0.00	0.01
$2^+(K^\pi = 1^+)$	3.35		0.02		0.09	0.11
$3^+(K^\pi = 1^+)$	3.23	0.00	0.01		0.12	0.13
$4^+(K^\pi = 1^+)$	4.60		0.01			0.01
$5^+(K^\pi = 1^+)$	6.31		0.36			0.36
D_α -fixed cal.						
$3^+(K^\pi = 3^+)$	1.58	0.12	0.01		0.00	0.13
$4^+(K^\pi = 3^+)$	2.95		0.41			0.41
$5^+(K^\pi = 3^+)$	4.27		1.07			1.07
$1^+(K^\pi = 1^+)$	2.11		0.00	0.10	0.00	0.11
$2^+(K^\pi = 1^+)$	3.61		0.01		0.41	0.42
$3^+(K^\pi = 1^+)$	3.88	0.19	0.15		0.51	0.85
$4^+(K^\pi = 1^+)$	6.77		0.01			0.01

plotted on the $(x, z) = (D_\alpha \sin \theta_\alpha, D_\alpha \cos \theta_\alpha)$ plane. The energy curves for $D_\alpha = 5$ fm are also shown as functions of θ_α . In the $D_\alpha \geq 5$ fm region, the contour of the energy surface on the (x, z) plane is deformed in the longitudinal ($\theta_\alpha = 0$) direction because of the prolate deformation of the ^{10}B cluster meaning that the α cluster at the fixed distance $D_\alpha = 5$ fm feels an attraction in the longitudinal direction. In other words, in the intrinsic system, the α cluster at $D_\alpha = 5$ fm energetically favors the longitudinal direction to form the linear 3α configuration rather than the transverse direction to form the triangle 3α configuration. In the $D_\alpha \leq 3$ fm region, the α cluster feels an effective repulsion in the longitudinal direction because of the Pauli blocking from the ^{10}B cluster, whereas it feels an attraction in the transverse ($\theta_\alpha = \pi/2$) direction.

In contrast to the intrinsic energy behavior, θ_α dependence of J^π -projected energy is not trivial because the energy is affected by not only potential energy but also kinetic energy of angular motion, i.e., rotational energy. Figure 7 shows energies of JK -projected states $\hat{P}_{MK}^{J^\pi} \Phi_{^{10}\text{B}(I_z)+\alpha}(D_\alpha, \theta_\alpha)$ at $D_\alpha = 5$ fm for $K = I_z$, which corresponds to the $L_{\alpha z} = 0$ projection. In high J states, the longitudinal direction ($|\theta_\alpha| \lesssim \pi/8$) is energetically favored than the transverse direction ($|\theta_\alpha - \pi/2| \lesssim \pi/8$) because of the larger moment of inertia (m.o.i.) of the longitudinal configuration than that of the transverse configuration for the $L_{\alpha z} = 0$ projection. However, in the lowest spin state ($JK = 11$), the energy almost degenerates in a wide region of θ_α because the kinetic energy is smaller for the transverse configuration than the longitudinal configuration because of the phase space factor $\sin \theta_\alpha$ in the $L_{\alpha z} = 0$ projection. This energy degeneracy results in the $L_\alpha = 0$ (S -wave) dominance in the $1^+(K^\pi = 1^+)$ state obtained by the D_α -fixed calculation.

Figures 8 and 9 show energies of JK -projected states at $D_\alpha = 5$ fm for $K \neq I_z$. Note that the $K \neq I_z$ projection corresponds to the $L_{\alpha z} \neq 0$ projection, and $K > I_z$ means the L_α alignment to the z direction (see Fig. 1(c)). For instance, the L_α -aligned state for $L_\alpha = 2$ (D -wave) is the $K = I_z + 2$ state. As shown in Figs. 8(a)-(c) and 9(a)-(d), L_α -aligned states energetically favor the transverse configuration because of the larger m.o.i. than that of the longitudinal configuration in the $L_{\alpha z} = 2$ projection. Figures 8 and 9 also show the α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ at $D_\alpha = 5$ fm in the $^{10}\text{B}+\alpha$ cluster states obtained by the D_α -fixed and full calculations. Let me first discuss the result obtained by the D_α -fixed calculation (Figs. 8(d)-(f) and 9(e)-(h)). In the $K^\pi = 3^+$ band states (Fig. 8(d)-(f)), the $J^\pi = 3^+$ state contains dominantly the longitudinal configuration ($|\theta_\alpha| \lesssim \pi/8$) rather than the transverse configuration ($|\theta_\alpha - \pi/2| \lesssim \pi/8$) as expected from the JK -projected energy curve for $K = I_z$. As J goes up to $J = 5$, the L_α -aligned component ($K = 5$) of the transverse configuration becomes large corresponding to the alignment of the orbital angular momentum L_α of the α cluster to $I_z = 3$ (the spin of (pn) cluster in the ^{10}B cluster). In the $K^\pi = 1^+$ band states (Fig. 9(e)-(h)), the $J^\pi = 1^+$ state shows the α -cluster probability distributed widely

in the $0 \leq \theta_\alpha \leq \pi/2$ region indicating the dominant $L_\alpha = 0$ (S -wave) component. As J increases, the longitudinal component becomes dominant compared with the transverse component. The alignment of L_α (the orbital angular momentum of the α cluster) and I_z is not so remarkable for $^{10}\text{B}(I_z^\pi = 1^+)$ differently from the $^{10}\text{B}(I_z^\pi = 3^+)$. Next, let me look into the result of the full calculation shown in Figs. 8(g)-(i) and 9(i)-(l). Compared with the D_α -fixed calculation, transverse components tend to be relatively more suppressed than longitudinal components in the full calculation. Note that the longitudinal ($\theta_\alpha = 0$) component is not dominant but is $30 \sim 40\%$, which is comparable to the $\theta_\alpha = \pi/4$ component. It indicates that $^{10}\text{B}+\alpha$ cluster states are different from the ideal linear configuration of a classical picture but they show significant quantum fluctuation in the angular (θ_α) motion and are regarded as the chain-like configuration that has relatively enhanced longitudinal components with suppressed transverse components.

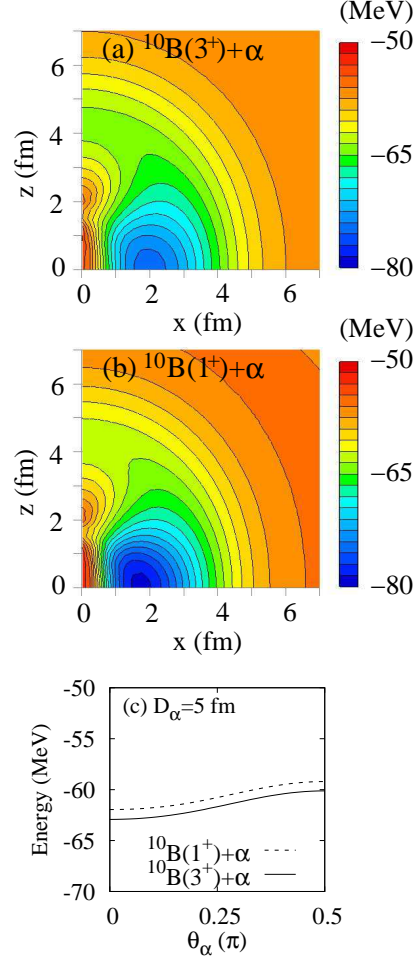


FIG. 6: Intrinsic energies of $^{10}\text{B}(I_z^\pi = 3^+) + \alpha$ and $^{10}\text{B}(I_z^\pi = 1^+) + \alpha$ before the parity and angular-momentum projections. Energies for (a) $^{10}\text{B}(I_z^\pi = 3^+) + \alpha$ and (b) $^{10}\text{B}(I_z^\pi = 1^+) + \alpha$ plotted on $(x, z) = (D_\alpha \sin \theta_\alpha, D_\alpha \cos \theta_\alpha)$, and (c) those at $D_\alpha = 5$ fm plotted as functions of θ_α .

The origin of the suppression of transverse components in $^{10}\text{B}+\alpha$ cluster states in the full calculation can be described by orthogonality to lower states which contain transverse components with $D_\alpha < 5$ fm. As shown in Fig. 6 for the energy surface on the $(D_\alpha, \theta_\alpha)$ plane, an energy pocket exists in the transverse direction ($\theta_\alpha \sim \pi/2$) around $D_\alpha \sim 2$, and therefore, transverse components contribute to low-lying ^{14}N states. Although the low-lying states are compact states containing mainly configurations with small D_α , transverse components with $D_\alpha = 5$ fm somewhat feed the low-lying states. As a result of the feeding of lower states, transverse components in the $^{10}\text{B}+\alpha$ cluster states near the threshold are suppressed. Figures 10 and 11 show the α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ for $\theta_\alpha = 0$ at $D_\alpha = 5$ fm and that for $\theta_\alpha = \pi/4$ and $\pi/2$ at $D_\alpha = 4$ fm. As seen in 10(a)-(c) for $^{10}\text{B}(I_z^\pi = 3^+) + \alpha$, the longitudinal ($\theta_\alpha = 0$) component of $^{10}\text{B}(I_z^\pi = 3^+) + \alpha$ shows the largest amplitude at the $K^\pi = 3^+$ band states (labeled by asterisks) and some fragmentation into neighboring states. Similarly, the longitudinal component of $^{10}\text{B}(I_z^\pi = 1^+) + \alpha$

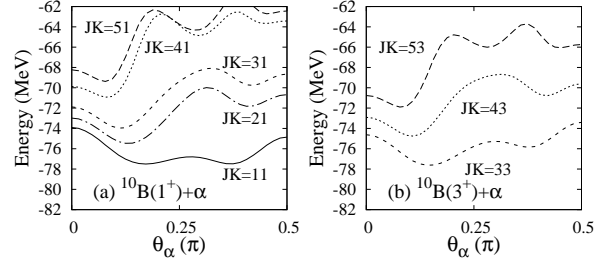


FIG. 7: Energies of the JK -projected $\Phi_{^{10}\text{B}(I_z^\pi)+\alpha}$ wave function $\hat{P}_{MK}^{J\pi} \Phi_{^{10}\text{B}(I_z^\pi)+\alpha}(D_\alpha, \theta_\alpha)$ with $K = I_z$ for (a) $^{10}\text{B}(I_z^\pi = 3^+)$ and (b) $^{10}\text{B}(I_z^\pi = 1^+)$. Energies for $D_\alpha = 5$ fm are plotted as functions of θ_α .

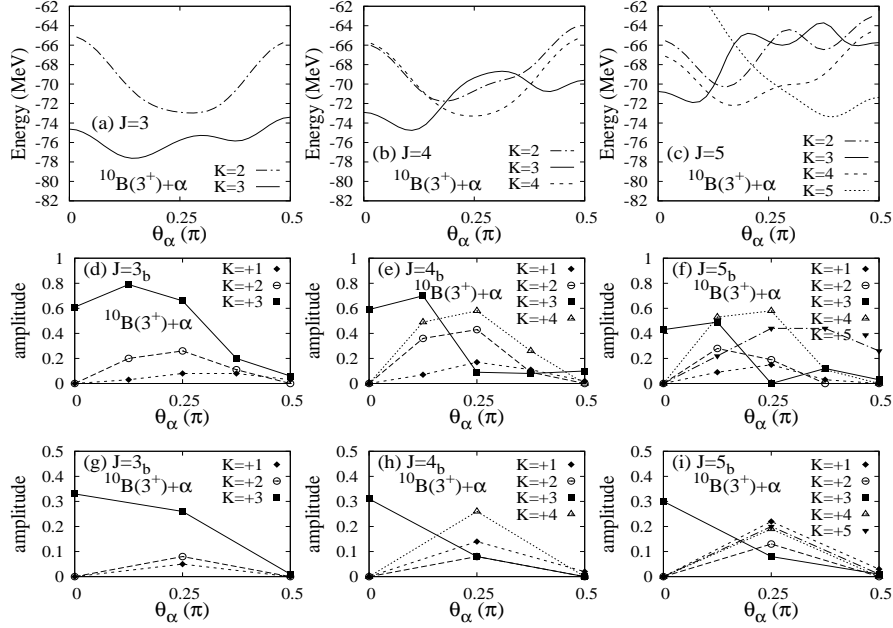


FIG. 8: (a)(b)(c) Energies of the JK -projected $\Phi_{^{10}\text{B}(I_z^\pi)+\alpha}$ wave function $\hat{P}_{MK}^{J\pi} \Phi_{^{10}\text{B}(I_z^\pi)+\alpha}(D_\alpha, \theta_\alpha)$ for $^{10}\text{B}(I_z^\pi = 3^+)$. (d)(e)(f) α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ for $I_z^\pi = 3^+$ at $D_\alpha = 5$ fm in $K^\pi = 3^+$ $^{10}\text{B} + \alpha$ cluster states obtained by the D_α -fixed calculation and (g)(h)(i) that obtained by the full calculation.

concentrates on the $K^\pi = 1^+$ band states (see Fig. 11(a)-(e)). On the other hand, transverse components feed states lower than $^{10}\text{B} + \alpha$ -cluster states as seen in Fig. 10(d)(f) and Fig. 11(f)(g)). Consequently the α cluster in $^{10}\text{B} + \alpha$ -cluster states near the threshold tends to avoid transverse configurations so as to satisfy orthogonality to lower states. This mechanism is consistent with the discussion of Ref. [54] for linear-chain 3α states in ^{14}C .

V. SUMMARY

I calculated positive-parity states of ^{14}N with the $^{10}\text{B} + \alpha$ cluster model and investigated $^{10}\text{B} + \alpha$ cluster states. Near the α -decay threshold energy, I obtained the $K^\pi = 3^+$ and $K^\pi = 1^+$ rotational bands having the developed α cluster with the $^{10}\text{B}(3^+)$ and $^{10}\text{B}(1^+)$ cores, respectively. I assigned the $3^+(K^\pi = 3^+)$ state in the present result to the experimental 3^+ at $E_r = 1.58$ MeV observed in α scattering reactions by ^{10}B , and showed that the calculated α -decay width agrees to the experimental width.

I analyzed the component of the longitudinal configuration having an α cluster in the longitudinal direction of the deformed ^{10}B cluster, which corresponds to a linear-chain 3α structure with valence nucleons. In the spectra

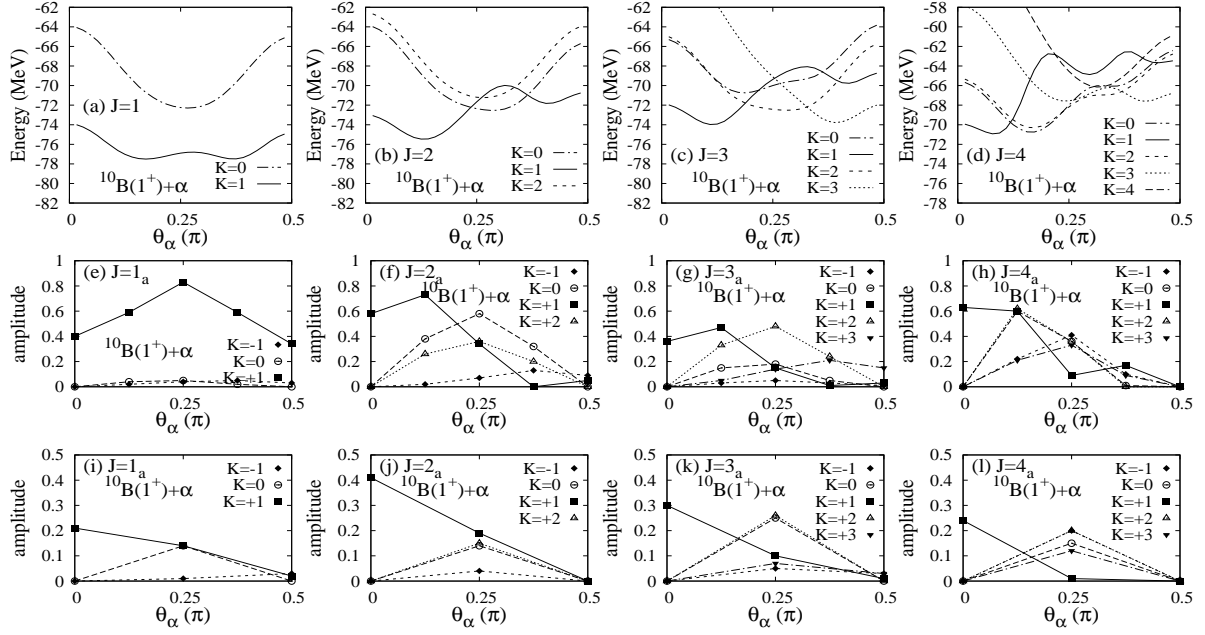


FIG. 9: (a)-(d) Energies of the JK -projected $\Phi_{10\text{B}(I_z^\pi)+\alpha}$ wave function $\hat{P}_{MK}^{J\pi} \Phi_{10\text{B}(I_z^\pi)+\alpha}(D_\alpha, \theta_\alpha)$ for $^{10}\text{B}(I_z^\pi = 1^+)$. (e)-(h) α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ for $I_z^\pi = 1^+$ at $D_\alpha = 5$ fm in the $K^\pi = 1^+$ $^{10}\text{B}+\alpha$ cluster states obtained by the D_α -fixed calculation and (i)-(l) that obtained by the full calculation.

of ^{14}N , the linear-chain component concentrates at the $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands. However, the $^{10}\text{B}+\alpha$ cluster states are different from the ideal linear configuration of a classical picture but they show significant quantum fluctuation in the angular (θ_α) motion and are regarded as the chain-like configuration that has relatively enhanced longitudinal components and suppressed transverse components. The orthogonality to low-lying states plays an essential role in the suppression of the transverse component.

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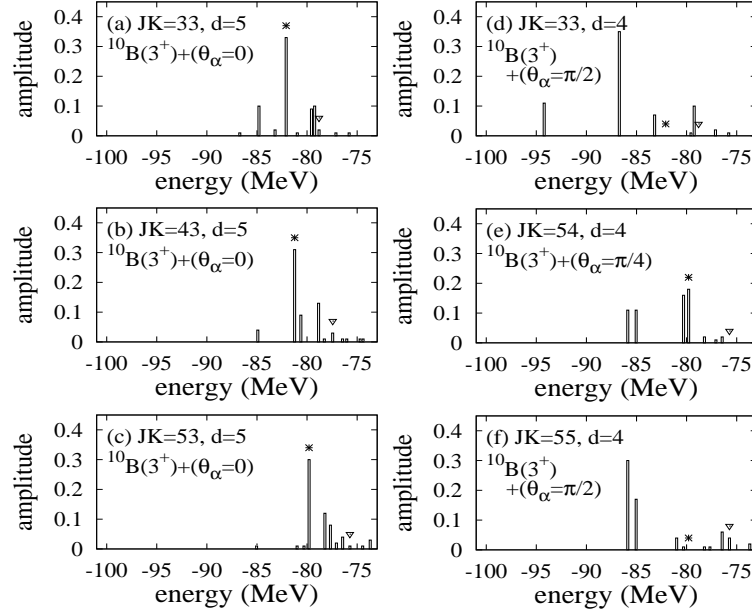


FIG. 10: α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi); D_\alpha, \theta_\alpha)$ for $I_z^\pi = 3^+$. D_α is taken to be $D_\alpha = 5$ fm for $\theta_\alpha = 0$, and $D_\alpha = 4$ fm for $\theta_\alpha = \pi/4$ and $\pi/2$. Asterisk and down-triangle symbols show $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, respectively.

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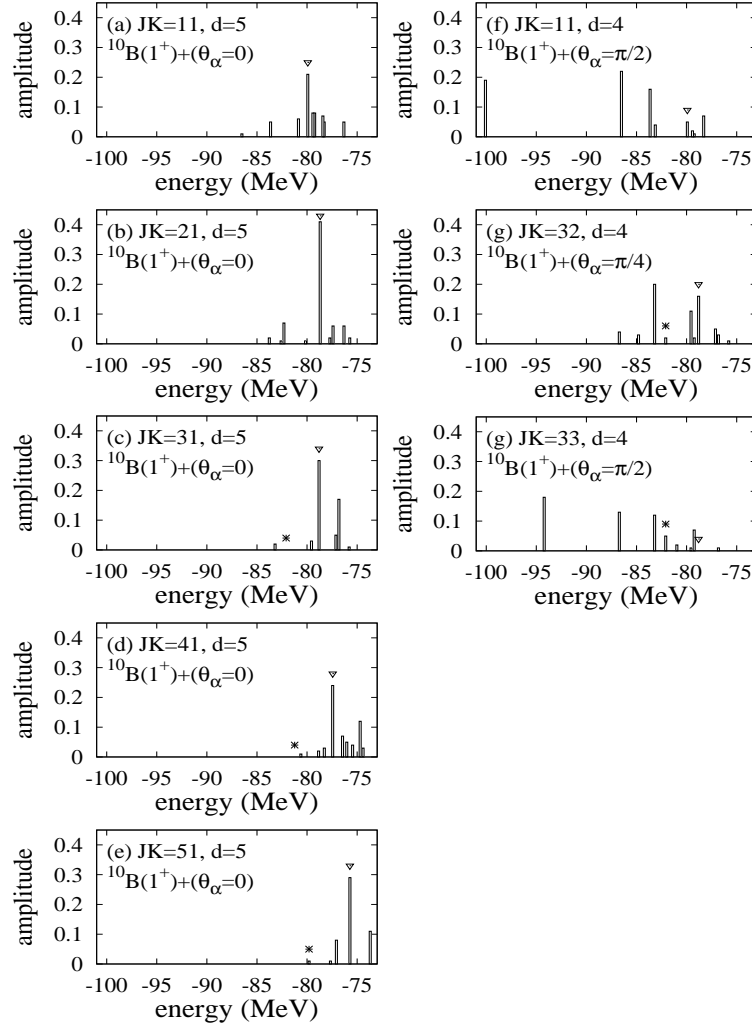


FIG. 11: α -cluster probability $P(JK; ^{10}\text{B}(I_z^\pi; D_\alpha, \theta_\alpha))$ for $I_z^\pi = 1^+$. D_α is taken to be $D_\alpha = 5$ fm for $\theta_\alpha = 0$, and $D_\alpha = 4$ fm for $\theta_\alpha = \pi/4$ and $\pi/2$. Asterisk and down-triangle symbols show $^{10}\text{B} + \alpha$ cluster states in the $K^\pi = 3^+$ band $K^\pi = 1^+$ bands, respectively.

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